Performance Based Learning and Assessment Task

Where Should We Sit?

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
Students will consider where to sit in a movie theatre based on optimal viewing angle and which seats are available. This will involve using the properties of inscribed angles. This is designed to be inquiry-based that progressively becomes guided depending on the students' needs throughout the task.

II. UNIT AUTHOR:
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III. COURSE:
Geometry

IV. CONTENT STRAND:
Circles and inscribed angles

V. OBJECTIVES:
The learner will be able to

- analyze a practical situation for applications of inscribed angles and their properties;
- consider multiple solution options, weighing pros and cons;
- use prior knowledge and investigate properties of geometric figures, including triangles, circles, and angles;
- communicate and represent their findings

VI. REFERENCE/RESOURCE MATERIALS:
Theatre diagram handout (attached), Assessment Rubric (attached), protractors, compasses, pencils, easel-size self-stick notes, markers, colored pencils, rulers.

VII. PRIMARY ASSESSMENT STRATEGIES:
Students will create poster representations of their findings using easel-sized self-stick notes, and then share these with the class and talk about what they did. Students will also self-assess with the provided rubric, which will also be used by the teacher as the evaluation rubric.

VIII. EVALUATION CRITERIA:
See assessment list on the attached rubric. An exemplary work benchmark is also attached.

IX. INSTRUCTIONAL TIME:
Two 55-minute class periods

Where Should We Sit?

Strand
Geometry

Mathematical Goals and Objective(s)
The overall goal of this task is for the learners, working in pairs, to consider an application of inscribed angles in the real world. This task is designed to be inquiry-based. By allowing the learners to move towards their own conception of inscribed angles, the learning becomes more organic and students can personalize the experience based on their knowledge of movie theatres. The presentation component of this task allows students to work on performance skills – communication, representation, organization of ideas – to describe their process to their peers.

Related SOL
G.11 – The student will use angles, arcs, chords, tangents, and secants to
a) investigate, verify, and apply properties of circles;
b) solve real-world problems involving properties of circles.

NCTM Standards
- Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Create and use representations to organize, record, and communicate mathematical ideas.

Materials/Resources
Theatre Diagram Handout (attached), Assessment Rubric (attached), protractors, compasses, pencils, easel-size self-stick notes, markers, colored pencils, rulers.

Assumption of Prior Knowledge
- Angles – measuring and classifications (acute/obtuse/right)
- Circles – measuring and parameters (center/radius)
- Triangles – angle sum theorem and classifications (scalene, isosceles, equilateral)
- How to use a protractor, compass, and ruler.
- Students should ideally be at Van Hiele level 3 (abstraction) in regards to angles, circles, or triangles. Students at lower Van Hiele levels may need the inquiry to be more structured for this task.
- Students may find it difficult to abstractly go from triangles to circles, thus realizing the relationship between inscribed angles. Since this task is intended to introduce the students to inscribed angles, confusion at the beginning is assumed. The teacher's role is to move between groups and identify/address misconceptions as they come up, as well as guide the inquiry where needed.

Introduction: Setting Up the Mathematical Task

- Ask the class: "Where is the best place to sit in a movie theatre?" and allow for some responses. This will yield the students' own preferences and the reasons for them.
- Ask students if they know what "viewing angle" means. Explain it is how much you move your head side-to-side when watching a screen.
    - Some students may connect with this if they have a big screen TV in their house in a small room.
- Tell the class that we are going to be figuring out where is the best place to sit in a movie theatre with a flat floor (*not* stadium seating).
- Students will be working in pairs. Once pairs are formed, hand out the Theatre Diagram Handout (attached).
- This is a guided inquiry task. Once students have had some time to ponder the prompt (the handout), the teacher should gradually guide the students in their investigative process, based on each group's needs.
- Guiding the inquiry can be accomplished by suggesting things like, "How can you connect the 'viewing angle' to the angles of a triangle?" or, "What about circles – could they help?"
- Inquiry is often a progressive and recursive process. The students should be bouncing ideas off the teacher throughout the investigation, as this dialogue will move the discovery along.
- At the end of the task: Students will conclude the task by organizing their information, creating a visual representation on easel-sized self-stick notes, and taking turns communicating their process and findings to the rest of the class.
- The task is expected to take two 55-minute class periods. The first class period would be for introducing the task and allowing for small group exploration (described below), as well as creating the visual representation component (easel-size self-stick notes) of the assessment. The second class period would give groups a few minutes to collect their thoughts and get ready, followed by each group taking turns presenting to the class.

Student Exploration

Individual Work

- After setting up the task (described above) and handing out the Theatre Diagram Handouts, students will spend a few minutes thinking about the problem on their own. This gives them time to pull prior knowledge and begin thinking about connections.
- Students will then be paired up for the Small Group and Sharing sections below. In addition to the Theatre Diagram Handout, students should have pencils, protractors, compasses, and rulers for this part of the task.

Small Group Work

- After students have been paired up, they should begin the inquiry process. The teacher should be moving between groups to check on progress and address any misconceptions that may arise.
- Since this is an inquiry-based task, the teacher should not give students an explicit direction to go in; rather, s/he should ask each group to describe their thinking. This also supports the communication objective of the task. In doing so, the teacher can make recommendations by questioning and commenting on what the students are saying. This is how the inquiry can be guided.
- Students should begin creating their visual representations on the easel-size self-stick notes. The teacher will make markers and colored pencils available for the students to use. Each group should plan on their presentation taking about 5 minutes.
- Students that finish quickly can move on to "Part II" on the handout, which is an extension activity. This involves deriving the relationship between inscribed angle measure and central angle measure. A benchmark example of this has also been provided at the end of this document. Students that

pursue this part of the task are encouraged to make an additional sticky note presentation for these findings. Depending on the level of the class, the teacher can decide whether or not to use Part II.

- As students are finishing up, hand out the Assessment Rubric so they can self-assess their performance, and check that they have completed all parts of the task. The teacher should collect these before class finishes so they can be handed back to students the next day to self-assess their presentation to their peers.

Whole Class Sharing/Discussion

The whole group sharing is intended for the second 55-minute class period. Give the groups about 5 minutes to get organized for their presentations. The teacher will also hand back the students' self-assessments from the previous class day, so they can self-assess their presentations. Each group will have 5 minutes to present to the class. The teacher will be using the Assessment Rubric to assess each group.

Student/Teacher Actions:

- Once paired up, students should be discussing with their partners the possibilities of investigation. They will have received the Theatre Diagram Handout and the teacher will have introduced the task by asking about prior knowledge of the viewing angle and students' preferred seat location when going to the movies.
- To facilitate learning, the teacher should be assisting the groups and questioning the students' thinking to move the inquiry along in a productive direction.

  Questions for students:
  - ✓ If points D, E, and F were on a circle, how could this be used to find a seat with the same viewing angle?
  - ✓ Can you find a relationship between the angle created where two chords meet on the edge of a circle and the arc measure created where the chords intercept the circle?
  - ✓ What relationship can you find between the central angle of a circle and an angle created by two chords which use the same points on the intercepted arc as the central angle?
- The teacher should be looking for misconceptions that have arisen through the process.

  Common Misconceptions:
  - ✓ Central angle and inscribed angle are congruent.
  - ✓ Inscribed angles are not congruent even if their chords share the same arc interception points.
  - ✓ Confusing the measure of an arc with the length of an arc.
  - ✓ Students may confuse the relationship between angles created by intersecting chords with the inscribed and central angles.
- The teacher should be prepared to close the task with a summary of the groups' findings. This should take place after all groups have presented.
- Ideally, the next class period after the task would be spent introducing inscribed angles and their properties.

Monitoring Student Responses

- The teacher should be:
  - o Moving between groups and asking each group to describe what they are doing and thinking;
  - o Questioning the learners as they are describing their methods:
    - ✓ How did you find the relationship you are discussing?
    - ✓ Would your findings apply to any circle regardless of size?
    - ✓ What was the reasoning behind your choosing the strategy you did?
- The learners should be:
  - o Discussing with their partners as to how to investigate the problem;
  - o Describing their thinking to their partner;
  - o Describing their thinking to the teacher; and
  - o Thinking about how they will summarize their process and findings to the class.

- The task will be summarized by the presentation/assessment component of the task.
    - The students will create visual representations of their process and findings, and present these to the class.
    - Afterwards, the teacher will close the task by summarizing the groups' findings and offering mathematical content to explain the problem.

      The summary will include:
        - ✓ Deriving the relationship between the central angles and inscribed angles of a circle (Those students able to get to part 2 of the assessment will have had a chance to do this for themselves).
        - ✓ Discussing other potential uses for inscribed angle relationships.

Assessment List and Benchmarks

- Assessment List and Evaluation Criteria are attached on pages 6 and 7 of this document.
- Teacher will complete the Assessment Rubric during the presentations (2^{nd} class period); student will use the same rubric to self-assess at the end of the inquiry (1^{st} class period).
- Students with fine motor skill accommodations will need to have a resource teacher help them.
- The visual component of the presentation will be assessed as part of the verbal presentation in front of the class (Rubric Item #9).
- Benchmark examples of work are included; they were created by the authors.

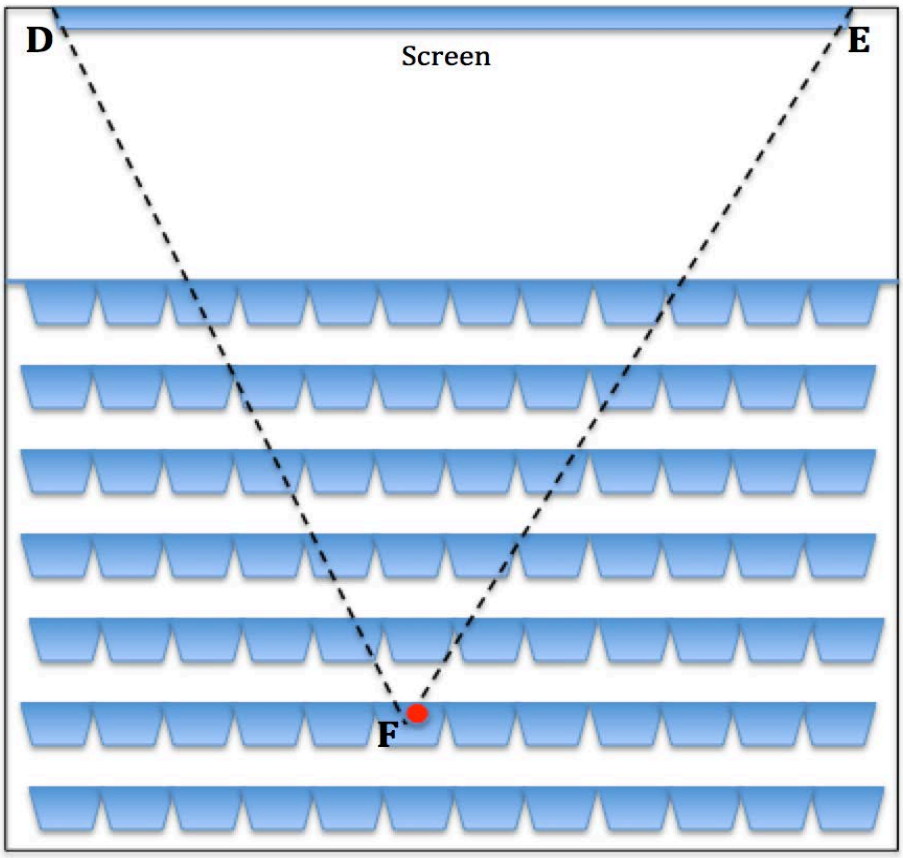
Geometry Name_____

Date_____ Pd._____

Where Should We Sit?

Theatre Diagram Handout

Where is the best place to sit in a movie theatre? Way up front? Way in the back? In the middle? What do you do if your favorite seat is taken? Look at this diagram of a typical movie theatre with a ***flat*** floor:



Your Task:

- Label the rows (*A* nearest the screen, through *G* at the back) and the seat numbers (starting with 1 on the far left of the diagram, and counting towards the right). Seats #1 will align with point D.
- Suppose you choose to sit in Row F, 6th seat from the left (point F in the diagram).
- Part I: Assume someone has already taken your seat… where else can you sit that will give you the same viewing angle?
    - Once you are done, get a big sticky note and come up with a presentation about what you did.
    - Each group will have 5 minutes to present their ideas to the class.
- Part II: Next, think about how you can calculate this viewing angle in terms of another angle (in an expression). The teacher will give you a hint if you need it. Make a 2nd sticky note to tell the class what you think. If unable to find how to measure the viewing angle, get the PART II handout from the teacher.

Geometry Name_____

Date_____ Pd._____

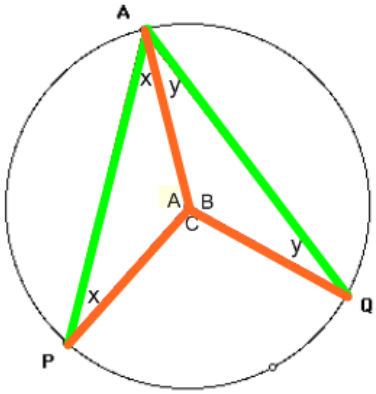
Where Should We Sit?

Part II – What is the viewing angle?

Now that you have determined where to sit, we should now ask, "What *is* the viewing angle?"

The figure below will help you in finding the measure of the viewing angle (Angle PAQ in the given diagram is the viewing angle).

Find an equation that represents angle PAQ in terms of angle C.



Angle A=180-2x
Angle B=180-2y
Angle C=360-Angle A-Angle B
Angle PAQ= x+y

Hint: Set angle PAQ equal to Angle C.

Where Should We Sit?
Assessment Rubric

Student Name_____
Partner Name_____

Assessment List

Rubric #	Rubric Item	Points possible	**Student** **(self-assessed)**	**Teacher**
Inquiry:				
1	Talked with teacher during investigation to discuss ideas and process	2	2	
2	Demonstrated knowledge of angles, circles, and triangles	2	2	
3	Worked collaboratively with partner	2	2	
4	Managed time well and completed within allotted class time	2	2	
5	Demonstrated critical thinking skills	2	2	
Presentation:				
6	Spoke clearly and projected voice during presentation	2	2	
7	Used appropriate vocabulary	1	1	
8	Managed time well (5 minute slot) and shared talking time with partner	2	2	
9	Created a visual component that aided in the communication of ideas	2	2	
10	Communicated their ideas and process effectively	2	2	
11	Answered questions from teacher and other students	1	1	

Total Score:_____20 / 20

Notes/Comments:

Rubric #	Rubric Item	**2 points**	**1 point**	**0 points**
Inquiry:				
1	Talked with teacher during investigation to discuss ideas and process	Student engaged in open and flowing discourse with teacher; discussed ideas thoroughly and was comfortable with uncertainty.	Student engaged in some discourse with teacher, but there was hesitation or discussion was forced. Teacher needed more from student.	Student did not engage in discourse with teacher.
2	Demonstrated knowledge of angles, circles, and triangles	Student shows proficiency in all these areas; was able to apply concepts to this task.	Student showed some proficiency, but was unsure about some relevant properties.	Student demonstrated no prior knowledge.
3	Worked collaboratively with partner	Student fostered a collaborative environment with partner; was open to hearing partner's ideas.	Student worked with partner, but was not open to partner's ideas and/or wasn't productive.	Student did not try to work collaboratively with partner.
4	Managed time well and completed within allotted class time	Student made consistent progress through inquiry and managed the class time well.	Student worked pretty well but had to be reminded to stay on task.	Student did not use class time well.
5	Demonstrated critical thinking skills	Student showed applied critical thinking throughout the inquiry.	Student showed some critical thinking skills, but did not apply them.	Student did not think critically.
Presentation:				
6	Spoke clearly and projected voice during presentation	Student spoke clearly, projected voice, and made presentation enjoyable.	Student spoke quietly or did not face the class while talking.	Student did not speak.
7	Used appropriate vocabulary	Not Applicable	Student used mathematics vocabulary appropriately.	Student did not use math vocabulary.
8	Managed time well (5 minute slot) and shared talking time with partner	Student managed time well and presented all information. Student and partner shared talk time equally and effectively.	Student got some information presented but ran out of time. Student and partner talk time was not evenly shared.	Student did manage time well. Student did all the talking, or did not talk.
9	Created a visual component that aided in the communication of ideas	Visual component of presentation was neat, organized, and effective in aiding the communication of ideas.	Visual component was sloppy or unorganized; did not effectively add to the communication of ideas during presentation.	Visual component was incomplete or missing.
10	Communicated their ideas and process effectively	Student communicated their ideas and process effectively; the presentation was engaging.	Student communicated some ideas, but something was missing from the presentation.	Student was ineffective in presenting their ideas.
11	Answered questions from teacher and other students	Not Applicable	Student answered (or attempted to answer) questions satisfactorily.	Student did not answer (or attempt to).

Benchmark Example of Student Work (next 4 pages)

Geometry

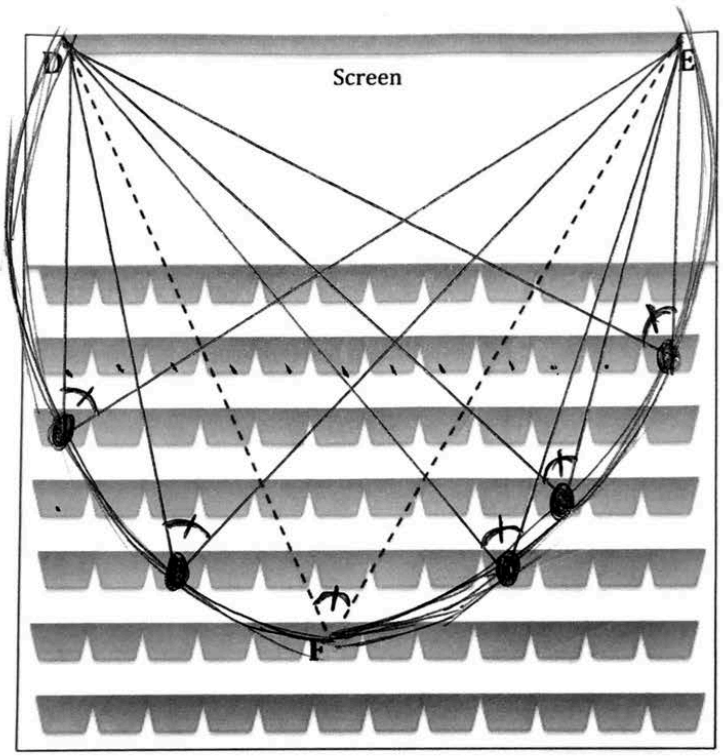
Name Benchmark Example

Date 9/27/15 Pd. ____

Where Should We Sit?

Theatre Diagram Handout

Where is the best place to sit in a movie theatre? Way up front? Way in the back? In the middle? What do you do if your favorite seat is taken? Look at this diagram of a typical movie theatre:



Your Task:

You could sit anywhere on the circle formed by points D, E, and F.

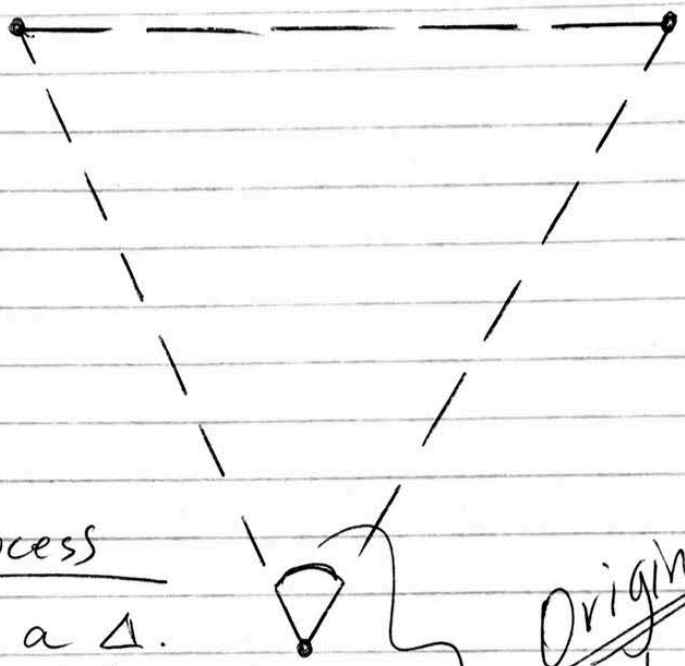
- Suppose you choose to sit 6th row back, 6th seat in from the left (point F in the diagram).
- Assume someone has already taken your seat… where else can you sit that will give you the same viewing angle? Any of the points shown would have the same viewing angle.
- Once you are done, get a big sticky note and come up with a presentation about what you did.
  - Each group will have 5 minutes to present their ideas to the class.

5

"Where Should We Sit?"

Benchmark Example of Student Work

BRAINSTORMING NOTES



Original Seat:
Need to keep this angle the same.

@ other seats:

Thought Process

- DEF is a Δ.
- All Δ Σ = 180°
- Any "other" Δ would change the viewing angle.
- Need a Δ that leaves the viewing angle the same and just changes the other two...

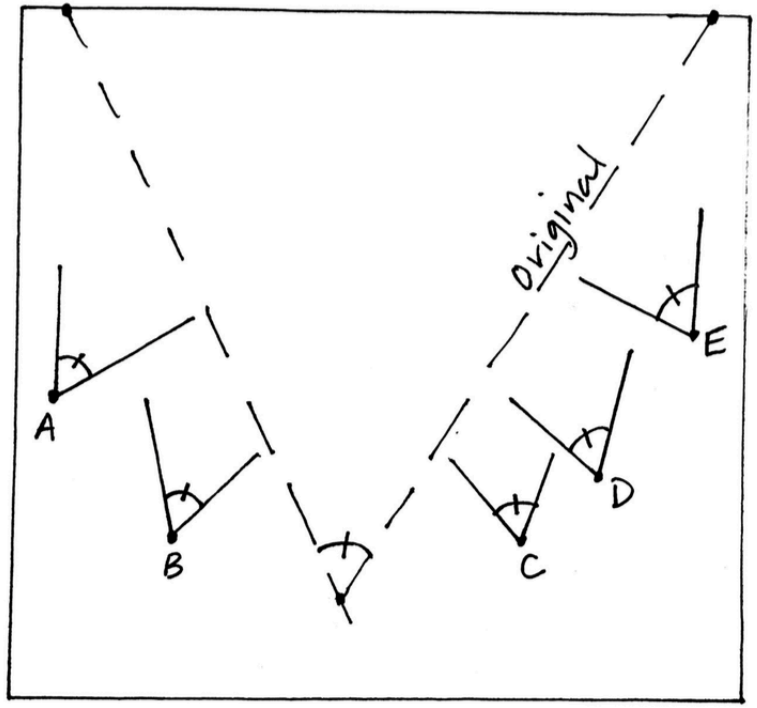
A-ha!

Circle - Would leave viewing angle the same and only change other two. (±)

Where Should We Sit?

Benchmark Example – Presentation "Sticky Note"

Theatre Diagram:



We found that by drawing a circle through the original 3 points, we could find other seats that laid on the circle as well. Any of these seats would give the same viewing angle as the original. This is called inscribed angles.

Original Seat: 6th Row, 6th Seat from (L)

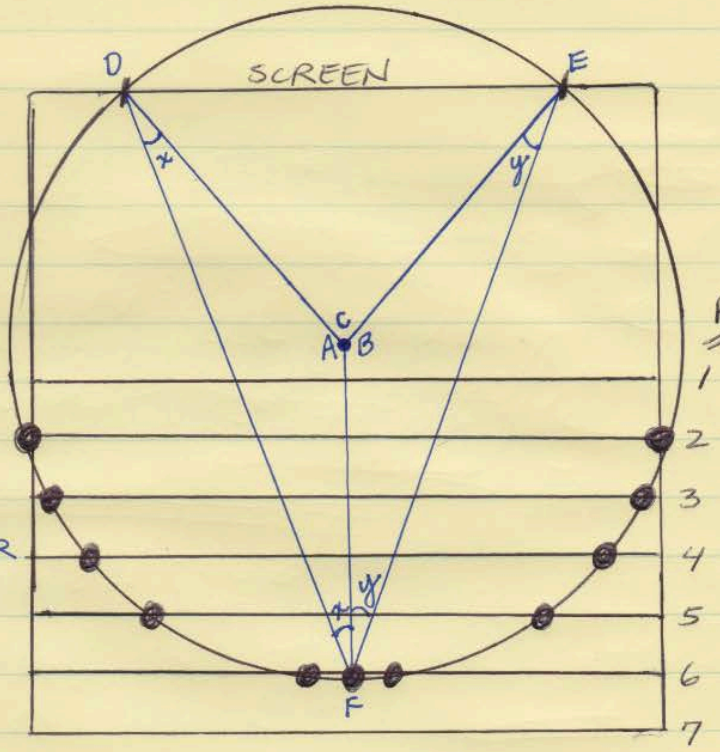
Alternative Seats:

A: 3rd Row, 1st seat from (L)
B: 5th Row, 3rd seat from (L)
C: 5th Row, 9th seat from (L)
D: 4th Row, 10th Seat from (L)
E: 2nd Row, 12th seat from (L)

Part II - Example Benchmark
"Where Should We Sit?"

KNOWN:

- POINT F IS THE PREFERRED SEAT.
- THE CENTER POINT OF THE CIRCLE IS SHOWN
- THESE TRIANGLES ARE ISOSCELES.
- WE CAN CALCULATE THE VIEWING ANGLE IN TERMS OF THE CENTRAL ANGLE "C" BY USING ALGEBRA.
- THIS CAN BE REPEATED FOR ANY OF THE "DOTS".



BOLD DOTS REPRESENT SEATS W/ APPROX. SAME VIEWING ANGLE

STATING THE OBVIOUS:

$\angle DEF = x + y$

$\angle A + 2x = 180^\circ$

$\angle B + 2y = 180^\circ$

$\angle A + \angle B + \angle C = 360^\circ$

APPLYING THESE STATEMENTS:

$\angle A = 180^\circ - 2x$ $\quad \angle B = 180^\circ - 2y$ $\quad \angle C = 360^\circ - \angle A - \angle B$

$\therefore \quad \angle C = 360^\circ - (180^\circ - 2x) - (180^\circ - 2y)$

$\angle C = \cancel{360^\circ} - \cancel{180^\circ} + 2x - \cancel{180^\circ} + 2y$

SIMPLIFYING GIVES US: $\angle C = 2x + 2y$

FACTORING: $\angle C = 2(x + y)$

SUBSTITUTING FOR $\angle DEF$: $\angle C = 2(\angle DEF)$

SOLVING FOR $\angle DEF$: $\left(\quad\right)\frac{1}{2}$

$\therefore$ THE VIEWING ANGLE IS HALF THE CENTRAL ANGLE

$$\frac{1}{2}\angle C = \angle DEF$$

PBA Rubric

The following rubric and category descriptions will be used to assess your PBA. Please review the rubric carefully. You will turn in your self-assessment with each draft of the project.

Number	Element	Point Value	Self-Graded	Teacher Grade
1	The Template is complete	8	8	
2	All materials and answer keys for the task are included	12	12	
3	The Template and materials are organized and structured	8	8	
4	The task is inquiry based	4	4	
5	The task is connected to the real world	6	6	
6	The task requires higher order thinking skills/levels of cognitive demand	4	4	
7	The task is based on the SOL's and NCTM Standards	6	6	
8	The assessment is provided and is followed both by students and teacher	12	12	
9	The benchmark of exemplary student work is included	6	6	
10	The document is professionally presented	6	6	
	Total	**72**	**72**	

Descriptions of Each Category

Points will be awarded for each item that is provided within the category.

1. **The Template is complete**
    a. All nine Roman Numerals on the title page are completed
    b. All bolded items in the body of the template are completed
    c. The included information for each component is accurate
    d. All instructions are removed from the template

2. **All materials and answer keys for the task are included**
    a. Any activity has a worksheet or related document that shows how the information will be shared with students
    b. The activity documents are well structured with limited grammatical errors
    c. The answer keys are provided for all activities
    d. The answer keys are accurate and well structured

3. **The Template and materials are organized and structured**
    a. The template and materials are easy to follow
    b. The template and materials have no grammatical errors
    c. The template and materials could be downloaded and used by another teacher with limited issues
    d. The template and materials are conceptually connected

4. **The task is inquiry based**
    a. The task focuses on students' exploring mathematical concepts.
    b. The tasks allows for students to build conceptual understanding.

5. **The task is connected to the real world**
    a. The tasks focuses on integrating a real world situation
    b. The task allows for students to connect the mathematical concepts to the real world situation
    c. The situation is not contrived, but is related to a literal understanding of a concept (see non-examples on D2L)

6. **The task requires higher order thinking skills/levels of cognitive demand**
    a. The task requires students to analyze, evaluate, and or create mathematical meaning
    b. The task is either at the cognitive demand level: Procedures with Connections or Doing Math

7. **The task is based on the SOL's and NCTM Standards**
    a. SOL's are provided
    b. NCTM Standards are provided
    c. The appropriate standards are chosen
    d. There are no apparent missing standards

8. **The assessment is provided and is followed both by students and teacher**
    a. The assessment is in the form of a rubric
    b. A description is provided for how points are assigned each level of the rubric
    c. All essential mathematical concepts are included on the rubric
    d. All graded components are built into the rubric
    e. Students are instructed to use the assessment
    f. Teachers are instructed to use the assessment

9. **The benchmark of exemplary student work is included**
    a. The benchmark shows how students would work each activity
    b. The benchmark shows how students would complete the assessment
    c. The benchmark shows exemplary work
    d. The benchmark represents the final product you would expect to be presented in your class

10. **The document is professionally presented**
    a. Font formatting (size, style, type) is consistent throughout the document
    b. Bullets and other itemized lists are consistently formatted
    c. Individual page layouts are complete, with no partial or incomplete pages